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THE EFFECT OF SENSITIVITY ON SIMPLY ORDERED SAFE-ARM STRATEGIES

Denis A. Silvia

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August 1986

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This report amplifies and extends the theory of safe-arming shown in Ballistic Research Laboratory Technical Report ARBRL-TR-02444, "The Worst-Case Mathematical Theory of Safe-Arming", by Denis A. Silvia, May 1984, by introducing a methodology for computing the effect of sensitivities on a s/a system's safety. This methodology permits the quantitative analysis of how the sensitivity of an s/a system's input variables can be manipulated to maximize s/a performance.

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Using the new methodology, a microcomputer program has been developed which permits easy analysis to see the effect of different sensitivity strategies, such as weak links - how many are best, how weak they should be, where they should be placed, etc.

Some suprising results are reported which will provide s/a designers with valuable insights into where design resources are best invested.

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I. INTRODUCTION

In BRL TR-02444, "The Worst-Case Mathematical Theory of Safe- Arming,"[1] various simple strategies were analyzed to show which ones are suitable for use in safe-arm devices. The most practical strategies seem to be the ones that use simple ordering. No matter what the overall system strategy, it has been proposed that the number of safe-arm inputs (or variables) needed could be reduced by making some variables more sensitive than others. If some accident should occur, properly chosen sensitivities would make the safe-arm variables function in a safe order.

II. DEFINING SENSITIVITY

It is necessary to first adopt a definition of sensitivity that is relevant to the safe/arm (s/a) strategy. Sensitivity has to be related to the system strategy used by an s/a device. Consider exactly how sensitivity is supposed to affect a simply ordered safe-arm device. In the Simple Ordering (S3) strategy, the only factor that determines a system event (s/a accident) is the sequential order of the system binary variables. This order must be altered to change the probability of a system event.

Sensitivity must be defined in terms of order. That is, more sensitive variables will respond to given levels of stress sooner than less sensitive variables.

Let: {x,y,z, . . .} be a set of independent s/a input variables with sensitivities X, Y, ..., and

let: P[x,y] be the probability of the event sequence "x followed by y (not necessarily in immediate succession)."

Then, it is useful if the sensitivity of the variable x, WITH RESPECT TO the variable y, is defined by:

$$P[x,y] = X/(X+Y). \tag{1}$$

If q is an input variable with sensitivity DEFINED as unity, then:

$$P[q,x] = [1/(1+X)] = 1/(X+1) \tag{2}$$

and

$$P[x,q] = [X/(X+1)], \tag{3}$$

so that

$$P[q,x] + P[x,q] = [1/(X+1)] + [X/(X+1)] = 1. \tag{4}$$

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1 Silvia, Denis A., "The Worst-Case Mathematical Theory of Safe-Arming." Ballistic Research Laboratory Technical Report #TR-02444, May 1984.

In general:

$$P[x,y] + P[y,x] = 1. \tag{5}$$

For a system of three variables:

$$P[x,y,z] = [X/(X+Y+Z)]\,[Y/(Y+Z)]\,[Z/Z].$$
$$= [X/(X+Y+Z)]\,[Y/(Y+Z)]. \tag{6}$$

The general definition of system sensitivity for a simply ordered system of n variables can be readily constructed:

Let {x1,x2,x3,...,xn} be a set of n s/a input variables with individual sensitivity weights X1, X2, ...,Xn, respectively.

Then:

$$P[x1,x2,...,xn] = [X1/(\sum_{i=1}^{n} Xi)]...[Xh/(\sum_{i=h}^{n} Xi)]...[Xn/Xn]. \tag{7}$$

The function defined in equation (7) is a physically reasonable definition of sensitivity for practical problems, since a stress which is increasing with time will force the most stress sensitive variables to fail first. In the remaining sections the definition of sensitivity in terms of order will be used to examine how sensitivity techniques can enhance S3 safe-arm strategies.

III. FITTING ORDERED SENSITIVITY INTO THE S3 STRATEGIES

Sensitivity does not change which sequences lead to a system event, but it does change the probability that any given sequence will occur due to a random set of events. The fact that different sequences have different probabilities of occurrence means that each of these sequences must be individually specified and evaluated. As defined in reference 1, an S3[I/J] strategy is a simply ordered safe/arm system strategy with J independent variables of which I or more must function in correct order to generate a safe/arm signal to detonate the warhead. The S3[N/N] strategy does not pose any problem, because there is only one sequence which can lead to a system event in this strategy.

Let: S = P{system event}.
Let: {x,y, . . .} be the solution sequence.
Then:

$$S = [X/(X+Y+ \ . \ . \ .)]\,[Y/(Y+Z+ \ . \ . \ .)] \ . \ . \ . \tag{8}$$

A number of sensitivity strategies have been solved in closed form for the S3[N/N] systems. They are discussed in the next section.

The S3[(N-1)/N] strategy is much more complicated. As shown in Reference 1, the solution sequences fall into three classes: I, II and III. The total number of solutions is given by N^2-2N+2. This means that for a system of 12 variables, 122 different sequences lead to a system event. The difference between the methodology of Reference 1 and that needed for variables of differing sensitivity is that when sensitivity strategies are used every sequence contributes a different weight to the system function probability and must therefore be individually evaluated.

Class I sequences are specified by recursively using the Class II and Class III formulae. The sequences in Classes II and III can be specified readily. The set of Class II sequences can be written as the rows of the matrix:

2 1 3 4 5 . . . N
2 3 1 4 5 . . . N
.
.
.
2 3 4 . . . . N 1,

where the digits represent the order of the variables.

The set of Class III sequences can be written:

3 1 2 4 5 . . . N
4 1 2 3 5 . . . N
.
.
.
N 1 2 3 4 . . (N-1)

A computer program for the S3[(N-1)/N] strategy has been written for the IBM PC microcomputer. The program is listed in the Appendix.

IV. RESULTS FOR S3[N/N] STRATEGIES

This section discusses S3[N/N] systems. The order sensitivity concept can be added to the equation for S3[N/N] systems to give closed form equations for several useful sensitivity strategies. The S3[(N-1)/N] strategy is more complex and the microcomputer program listed in the Appendix will be used in the next section to explore numerically the effect of sensitivity on S3[(N-1)/N] systems.

There are two variables related to sensitivity that can be manipulated: range and distribution. The range is set by the highest and lowest (generally unity) sensitivity values in the strategy, while distribution determines how the range is alloted among the system variables. A strategy without sensitivity structuring is treated as a special case where all the variables have the same probability weight, i.e. a level distribution. It does not matter what sensitivity weight is used in a level sensitivity strategy since the same answer is obtained no matter what weight is chosen.

This is easily shown:

Let: k,...k, be a level distribution of N variables and s be the probability of a system failure.

Using Equation 7:

S = [k/Nk] [k/(N-1)k] . . . = 1/N!, no matter what value k assumes. (9)

A simple linear strategy is one in which the sensitivity weight starts at unity and increases by a constant number of units with each succeeding variable. S3[N/N] systems using this sensitivity strategy can also be written in closed form:

Let: 1, 2, 3, . . . be a simple linear strategy in N variables.

Then the weight of each variable W_i, is L and:

$$S = \left[1/\sum_{i=1}^{N} i\right]\left[2/\sum_{i=2}^{N} i\right] \cdots \left[N/\sum_{i=N}^{N} i\right] = N!/\left\{\left[\frac{N(N+1)}{2}\right]\left[\frac{(N-1)(N+2)}{2}\right] \cdots \left[\frac{2(N+(N-1))}{2}\right]\left[\frac{(1)(N+N)}{2}\right]\right\} = \frac{N!2^N}{N!\,(2N)!/N!} = \frac{N!2^N}{(2N)!} \quad (10)$$

Table 1. Level Vs. Simple Linear Strategies.

N	PROBABILITY WEIGHTS	P[SYSTEM EVENT]
1	(any)	1.0
2	1,1	.5
	1,2	.3
3	1,1,1	1.67E-1
	1,2,3	6.7E-2
5	1,1,1,1,1	8.3E-3
	1,2,3,4,5	1.0E-3
10	1,1,...,1	2.8E-7
	1,2,...,10	1.5E-9

It is clear that the linear sensitivity strategy is superior to the level one, especially for larger values of N. This is deceptive, however, because the larger values of N have a larger range of variable sensitivity weights. If the range of the linear strategies of Table 1 is equalized, the results are more representative. The equalized linear strategy can be written in closed form also:

Let: 1, 2, W be a linear distribution of N variables and maximum sensitivity, W.

Let: $A(i) = 1 + [(i-1)(W-1)]/(N-1)$ be the ith term in an equalized linear distribution corresponding to the ith term in the linear distribution above.

Then the equalized system event probability can be written:

$$S = \sum_{j=1}^{N} \{A(j)/[\sum_{k=j}^{N} A(k)]\}. \qquad (11)$$

Table 2. Equalized Linear Stategies.

N	PROBABILITY WEIGHTS	P[SYSTEM EVENT]
1	10	1.0
2	1,10	9.1E-2
3	1,5.5,10	2.1E-2
4	1,4,7,10	3.6E-3
5	1,3.25,5.5,7.75,10	4.6E-4
6	1,2.8,4.6, . . .,10	4.9E-5
7	1,2.5,4.0, . . .,10	4.4E-6
8	1,2.28,3.56, . . .,10	3.5E-7
9	1,2.125,3.25, . . .,10	2.4E-8
10	1,2,...,10	1.5E-9

A summary comparison of the three kinds of sensitivity strategy shown in Tables 1 and 2 is given in Table 3:

Table 3. Comparison of Level and Linear Strategies.

N	LEVEL	SIMPLE LINEAR	EQUALIZED LINEAR
1	1.0	1.0	1.0
2	0.5	3.3E-1	9.1E-2
3	1.67E-1	6.7E-2	2.1E-2
4	4.2E-2	9.5E-3	3.6E-3
5	8.3E-3	1.0E-3	4.6E-4
6	1.4E-3	9.6E-5	4.9E-5
7	2.0E-4	7.4E-6	4.4E-6
8	2.5E-5	4.9E-7	3.5E-7
9	2.8E-6	2.9E-8	2.4E-8
10	2.8E-7	1.5E-9	1.5E-9

Examination of Table 3 shows that the equalized linear strategy can achieve a system event probability of less than 1.0E-6 with only eight variables - two less than a level strategy. Even fewer variables are needed with a "weak link" sensitivity strategy. The weak link approach is commonly used in safety design where a chain of events is forced to fail at a predetermined place by making one link in the chain much more likely to fail than the other links. The methodology developed in this report is ideal for

examining the weak link strategy. One of the first questions is, "Where do we place the weak link?" As before, let the range be equalized to ten. Then Tables 4a, 4b, 4c and 4d show the effect of link location for several S3[N/N] systems:

Table 4a. Weak-Link Strategies for Several Values of N. N=2

N	LINK STRATEGY		P[SYSTEM EVENT]
2	10	1	9.1E-1
	1	10	9.1E-2

Table 4b. Weak-Link Strategies for Several Values of N. N=3

N	LINK STRATEGY			P[SYSTEM EVENT]
3	10	1	1	4.2E-1
	1	10	1	7.6E-2
	1	1	10	7.6E-3

Table 4c. Weak-Link Strategies for Several Values of N. N=5

N	LINK STRATEGY					P[SYSTEM EVENT]
5	10	1	1	1	1	3.0E-2
	1	10	1	1	1	9.2E-3
	1	1	10	1	1	2.3E-3
	1	1	1	10	1	4.2E-4
	1	1	1	1	10	4.2E-5

Table 4d. Weak-Link Strategies for Several Values of N. N=10

N	LINK STRATEGY										P[SYSTEM EVENT]
10	10	1	1	1	1	1	1	1	1	1	1.4E-6
	1	10	1	1	1	1	1	1	1	1	7.2E-7
	1	1	10	1	1	1	1	1	1	1	3.4E-7
	1	1	1	10	1	1	1	1	1	1	1.5E-7
	1	1	1	1	10	1	1	1	1	1	6.0E-8
	1	1	1	1	1	10	1	1	1	1	2.1E-8
	1	1	1	1	1	1	10	1	1	1	6.6E-9
	1	1	1	1	1	1	1	10	1	1	1.6E-9
	1	1	1	1	1	1	1	1	10	1	3.0E-10
	1	1	1	1	1	1	1	1	1	10	3.0E-11

It is obvious from Tables 4 that the weak link strategy is superior to the linear one and that the best location for a weak link is at the end. It is not clear, however, whether another strategy might be better. Is one weak link enough? Two? It is likely that the best number of links depends on the number of variables. Equation 11 can modified to include this strategy also:

Let: 1, 1,...,W,...,W be a weak link strategy of N variables with k links of weight W.

Then:

$$S = (kW)!/\{[kW+(N-k)]!k!\} \qquad (12)$$

Tables 5 explore this equation for different numbers of links of weight 10:

Table 5a. Optimal Weak-Link Strategies for Several Values of N. N = 2

N	LINK STRATEGY	P[SYSTEM EVENT]
2	1 10	9.1E-2
	10 10	1.0

Table 5b. Optimal Weak-Link Strategies for Several Values of N. N = 3

N	LINK STRATEGY	P[SYSTEM EVENT]
3	1 1 10	7.6E-3
	1 10 10	2.4E-2
	10 10 10	8.3E-1

Table 5c. Optimal Weak-Link Strategies for Several Values of N. N = 5

N	LINK STRATEGY	P[SYSTEM EVENT]
5	1 1 1 1 10	4.2E-5
	1 1 1 10 10	4.7E-5
	1 1 10 10 10	1.7E-4
	1 10 10 10 10	1.0E-3
	10 10 10 10 10	8.3E-3

Table 5d. Optimal Weak-Link Strategies for Several Values of N. N = 10

N	LINK STRATEGY										P[SYSTEM EVENT]
10	1	1	1	1	1	1	1	1	1	10	3.0E-11
	1	1	1	1	1	1	1	1	10	10	4.0E-12
	1	1	1	1	1	1	1	10	10	10	3.2E-12
	1	1	1	1	1	1	10	10	10	10	6.2E-12
	1	1	1	1	10	10	10	10	10	10	9.1E-11
	1	1	1	10	10	10	10	10	10	10	5.3E-10
	1	1	10	10	10	10	10	10	10	10	3.7E-9
	1	10	10	10	10	10	10	10	10	10	3.0E-8
	10	10	10	10	10	10	10	10	10	10	2.3E-5

It is clear from Tables 5a-5d that the number of variables does determine the optimum number of weak links. Table 6 shows the optimum number of links for up to 10 variables.

Table 6. Summary of Optimal Strategies.

N	OPTIMUM NUMBER	P[SYSTEM EVENT]
2	1	9.1E-2
3	1	7.6E-3
4	1	5.8E-4
5	1	4.2E-5
6	2	2.0E-6
7	2	7.8E-8
8	2	3.0E-9
9	2	1.1E-10
10	3	3.2E-12

As Table 6 shows, an optimal weak link strategy is superior to any level or linear strategy for the same number of variables. The optimal weak link strategy of seven variables is even superior to a level strategy of ten variables. The strategy of seven variables with two weak links is significantly better than the 1/million requirement, so it is of interest to determine how much the sensitivity range can be reduced before the 1.0E-6 limit is reached.

Table 7. S3[N/N] STRATEGY REQUIREMENTS TO MEET A 1/MILLION SAFETY STANDARD.

WEAK LINK SENSITIVITY WEIGHT	NUMBER OF VARIABLES	NUMBER OF WEAK-LINKS	PROBABILITY OF SYSTEM EVENT
1	10	3	2.8E-7
2	9	3	2.5E-7
3	8	3	6.9E-7
4	8	3	2.2E-7
5	8	2	8.6E-8
6	7	2	6.7E-7
7	7	2	3.6E-7
8	7	2	2.0E-7
9	7	2	1.2E-7
10	7	2	7.8E-8

As Table 7 shows, we can use an optimal weak link strategy with seven variables and a maximum sensitivity level of only six to meet the 1/million requirement. The lower sensitivity in an optimal weak link strategy has other advantages, such as lower probability that the safe/arm will be inactivated by sensitive variables that function prematurely. This results in a "dud" munition.

V. RESULTS FOR S3[(N-1)/N] STRATEGIES

S3[(N-1)/N] system strategies are more complicated than S3[N/N] Strategies, but they offer lower dud rates. The microcomputer program listed in the Appendix has been used to explore the effect of sensitivity strategies on S3[(N-1)/N] systems.

Table 8 shows a comparison of level, simple linear and equalized linear strategies for 1 to 12 system variables:

Table 8. Comparison of Level and Linear Strategies.

N	LEVEL	SIMPLE LINEAR	EQUALIZED LINEAR
1	1.0E0	1.0E0	1.0E0
2	1.0E0	1.0E0	1.0E0
3	8.3E-1	6.7E-1	4.9E-1
4	4.2E-1	2.0E-1	1.3E-1
5	1.4E-1	3.8E-2	2.4E-2
6	3.6E-2	5.4E-3	3.5E-3
7	7.3E-3	6.0E-4	4.2E-4
8	1.2E-3	5.4E-5	4.2E-5
9	1.8E-4	4.2E-6	3.7E-6
10	2.3E-5	2.8E-7	2.8E-7
11	2.5E-6	1.7E-8	1.9E-8
12	2.5E-7	8.8E-10	1.2E-9

Once again the linear sensitivity strategies require a smaller number of variables than level ones for a given level of safety, so we can follow the pattern of the previous section and examine weak-link sensitivity strategies. Tables 9 show the event probabilities for several S3[(N-1)/N] systems using a single weak-link sensitivity strategy.

Table 9a. Weak-Link Strategies for Selected Values of N. N = 3

N	STRATEGY	P[SYSTEM EVENT]
3	10 1 1	9.9E-1
	1 10 1	9.2E-1
	1 1 10	5.8E-1

Table 9b. Weak-Link Strategies for Selected Values of N. N = 5

N	STRATEGY	P[SYSTEM EVENT]
5	10 1 1 1 1	3.4E-1
	1 10 1 1 1	1.8E-1
	1 1 10 1 1	9.4E-2
	1 1 1 10 1	5.5E-2
	1 1 1 1 10	4.3E-2

Table 9c. Weak-Link Strategies for Selected Values of N. N = 10

N	LINK STRATEGY	P[SYSTEM EVENT]
10	10 1 1 1 1 1 1 1 1 1	1.0E-4
	1 10 1 1 1 1 1 1 1 1	5.5E-5
	1 1 10 1 1 1 1 1 1 1	2.9E-5
	1 1 1 10 1 1 1 1 1 1	1.5E-5
	1 1 1 1 10 1 1 1 1 1	8.2E-6
	1 1 1 1 1 10 1 1 1 1	4.9E-6
	1 1 1 1 1 1 10 1 1 1	3.5E-6
	1 1 1 1 1 1 1 10 1 1	3.0E-6
	1 1 1 1 1 1 1 1 10 1	2.8E-6
	1 1 1 1 1 1 1 1 1 10	2.8E-6

As in the previous section, a weak-link is most effective in the last position of the variable sequence. Table 10 lists the system event probabilities for up to twelve variables with a sensitivity strategy using a single weak-link of weight 10 in the last variable position.

Table 10. Single Weak-Link Strategies.

N	P[SYSTEM EVENT]
1	1.0E0
2	1.0E0
3	5.8E-1
4	1.8E-1
5	4.3E-2
6	8.5E-3
7	1.4E-3
8	2.0E-4
9	2.5E-5
10	2.8E-6
11	2.8E-7
12	2.5E-8

In Section IV it was found that with larger sets of variables, system safety is improved with more than a single weak link. Table 11 lists the number of weak links and corresponding system event probabilities for systems up to 12 variables.

Table 11. Optimal Weak-Link Strategies.

N	OPTIMUM NUMBER OF LINKS	P[SYSTEM EVENT]
3	2	5.7E-1
4	2	5.5E-2
5	2	4.0E-3
6	2	2.5E-4
7	2	1.4E-5
8	2	7.7E-7
9	3	3.3E-8
10	3	1.1E-9
11	3	3.6E-11
12	3	1.1E-12

Table 11 shows that an optimal weak-link strategy with sensitivity weight ten requires only eight variables to provide protection at the 1.0E-6 level. This compares favorably with the 12 variables needed to meet the same criterion with a level strategy.

Table 12. S3[(N-1)/N] STRATEGY REQUIREMENTS TO MEET A 1/MILLION SAFETY STANDARD.

WEAK-LINK SENSITIVITY WEIGHT	NUMBER OF VARIABLES	NUMBER OF WEAK-LINKS	PROBABILITY OF SYSTEM EVENT
1	12	N/A	2.5E-7
2	11	4	1.9E-7
3	10	3	4.4E-7
4	10	3	1.2E-7
5	9	3	7.3E-7
6	9	3	3.3E-7
7	9	3	1.7E-7
8	9	3	9.2E-8
9	9	3	5.3E-8
10	8	2	7.7E-7

Table 12 shows how the sensitivity weight of the weak link variable(s) can affect the number of system variables needed to meet the 1/million safety standard.

VI. CONCLUSIONS

Order Sensitivity is a powerful concept that extends the Worst-Case safe/arm hypothesis to the analysis of more complicated and realistic safe/arm designs.

Order Sensitivity strategies can be incorporated into simply ordered safe/arm devices with fruitful results.

The Optimal Weak-Link sensitivity strategy is the best of those tested for both S3[N/N] and S3[(N-1)/N] systems.

Using the Optimal Weak-Link sensitivity strategy the number of variables needed to meet or better the 1.0E-6 safety standard can be reduced from 10 to 7 for an S3[N/N] system and from 12 to 8 for an S3[(N-1)/N] one.

The Ordered Sensitivity approach could be applied to other safe/arm strategies and, like the worst-case hypothesis for safe/arming, should be useful for general use in safety analysis and design. The discovery that the optimal number of weak links is dependent on the number of variables in a simply ordered safety system may have great significance in the design of safety.

APPENDIX

A MICROCOMPUTER PROGRAM TO COMPUTE SYSTEM EVENT PROBABILITIES FOR (N-1)/N SAFE-ARM DEVICES USING SIMPLE ORDERING + SENSITIVITY STRATEGIES

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A MICROCOMPUTER PROGRAM TO COMPUTE SYSTEM EVENT PROBABILITIES FOR (N-1)/N SAFE-ARM DEVICES USING SIMPLE ORDERING + SENSITIVITY STRATEGIES.

The S3[(N-1)/N] safe-arm system strategy can be readily solved in closed form if the system variables have a level sensitivity distribution, but if the system variables are not all of the same sensitivity then each sequence leading to a system event must be evaluated individually. The method used in this report is the same one described in Reference (1), Appendix B:

The set of solutions that lead to a system event is partitioned into three classes:

Class I consists of sequences in which the variable that is supposed to function first does function first.

Class II consists of sequences in which the variable that is supposed to function second functions first.

Class III consists of sequences in which one of the variables other than those that are supposed to function first or second functions first.

Class I sequences are enumerated indirectly. If variable #1 does function first, then no out-of-order has occurred. This means that the remaining N-1 variables are still permitted one out-of-order variable. But this is precisely the definition of an S3[(N-2)/(N-1)] strategy in the variables 2 to N. The Class I sequences can thus be found recursively:

Step 1. Variable #1 is assumed to function first.

Step 2. Variables 2-N are re-labeled 1', ..., (N-1)', respectively.

Step 3. Class II and III sequences, are enumerated for the strategy formed by variables 1' to (N-1)'.

Step 4. Steps 1 to 4 are repeated for the primed system (variable #1 is replaced by variable #1').

Sequences in Class II can be enumerated by inspection. If variable #2 functions first, then the single malfunction permitted by the strategy has already occurred. Variables 3 to N must then be in sequence. The only remaining variable is #1. There are N-1 possible positions for #1 in the sequence. The set of Class II sequences can be shown as the rows in the matrix:

$$
II = \begin{matrix}
2\ 1\ 3\ 4\ \ldots\ N \\
2\ 3\ 1\ 4\ \ldots\ N \\
2\ 3\ 4\ 1\ \ldots\ N \\
\vdots \\
2\ 3\ 4\ \ldots\ N\ 1
\end{matrix}
$$

Sequences in Class III are also easily enumerated. For each of the choices 3 to N for the first variable in the sequence, the one allowed misfunction has already occurred - just as in Class II. This means that all other variables must function in order. There are N-2 possible choices for the first variable and the set of Class III sequences can be shown as the rows in the matrix:

$$
III = \begin{matrix}
3\ 1\ 2\ 4\ \ldots\ N \\
4\ 1\ 2\ 3\ \ldots\ N \\
5\ 1\ 2\ 3\ \ldots\ N \\
\vdots \\
N\ 1\ 2\ 3\ \ldots\ (N-1)
\end{matrix}
$$

The evaluation process described above has been written into a program for the IBM PC Microcomputer. Although Basic is an unstructured language, some structuring can be introduced by using line number groups and "top down" programming techniques. The top level program is followed by the detailed listing of the program in IBM PC Basic.

TOP LEVEL PROGRAM

```
****NUSTART 6/13/1100****

INITIALIZATION (LINES 1-199)
  Input # Vairables (N)
  Dimension Arrays
  Input weight of each variable
ENDINITIALIZATION

MAIN PROGRAM (LINES 200-999)
  LOOP FROM I0=0 TO N-1
  Fill ZS Array with 99's
  Get the Class II sequences
  Fill ZS Array with 99's
  Get Class III Sequences
  Print Result
ENDMAIN PROGRAM

SUBROUTINE FILL IN ZS'S (Lines 1000-1999)
  Fill the ZS Array with 99's
ENDSUBROUTINE

SUBROUTINE 2'S (Lines 2000-3000)
  Re-label variables I0+1 to N as variables 1 to N-10
  LOOP to Construct Class II sequences
  Construct a Class II sequence
  GOSUB 5000 (Compute sequence probability)
ENDLOOP

SUBROUTINE 3'S (lines 3000-5000)
  LOOP to Construct Class III sequences
  Construct a Class III Sequence
  GOSUB 5000 (Compute sequence probability)
  ENDLOOP
ENDSUBROUTINE

SUBROUTINE 5000 (lines 5000-5999)
  PRINT System Strategy
  GOSUB 6000 (Computation)
ENDSUBROUTINE

SUBROUTINE 6000 (Lines 6000-6999)
  Compute sequence probability and add to System
  PRINT
ENDSURROUTINE
```

DETAILED PROGRAM LISTING

```
2 PRINT"****** NUSTART 6/13/1100******"
3 PRINT"":
4 DEFDBL A-H, L-N, O-Z
5 DIM ZS (50), NS (50)
6 PRINT, "INPUT N";:INPUT N
7 PROB=0:SUMWT=1:TERM=1
8 FOR I=1 TO N
9   PRINT, "INPUT WT. OF DET #;I;"  ";:INPUT NS(I)
10 NEXT I
12 FOR L%=1 TO N:PRINT NS(L%);:NEXT L%
13 PRINT""
197 REM
198 REM END INITIALIZATION
199 REM*************************************************************************
200 REM MAIN LOOP
201 REM
210 FOR I0=0 TO
230   GOSUB 1000:GOSUB 2000
240   GOSUB 1000:GOSUB 3000
270 NEXT I0
990 FOR I=1 TO N:ZS(I)=NS(I):NEXT I:GOSUB 5000
992 PRINT "N=";N;"   NS(I)=";:FOR K=1 TO N:PRINT NS(K);:NEXT K
993 PRINT"   PROB=";PROB:PRINT""
994 END
997 REM END MAIN LOOP
999 REM*************************************************************************
1000 REM SUBROUTINE FILL IN ZS'S
1001 REM
1010 FOR I1=0 TO N
1020   ZS(I1)=99
1030 NEXT I1
1990 RETURN
1998 REM END SUBROUTINE FILL IN ZS'S
1999 REM *************************************************************************
2000 REM SUBROUTINE 2'S
2020 FOR I2 =I0+1 TO N
2025   IF I2=I0+1 THEN 2950
2030   FOR J2=I0+1 TO I2-1
2040     ZS(J2) = NS(J2+1)
2050   NEXT J2
2060   ZS (I2)=NS(I0+1)
2065   IF I2=N THEN 2940
2070   FOR K2=I2+1 TO N
2080     ZS(K2)=NS(K2)
2090   NEXT K2
2940   GOSUB 5000
2950 NEXT I2
2980 REM
2990 RETURN
2998 REM END SUBROUTINE 2'S
2999 REM*************************************************************************
```

```
3000 REM SUBROUTINE 3'S
3005 IF I0>N-3 THEN 4020
3010 FOR I3=I0+3 TO N
3020   ZS(I0+1) = NS(I3)
3030   FOR J3 = I0+2 TO I3
3040   ZS(J3) = NS(J3-1)
3050 NEXT J3
3060 IF I3=N THEN 4000
3070 FOR J3=I3+1 TO N
3080 ZS(J3)=NS(J3)
3090 NEXT J3
4000 GOSUB 5000
4010 NEXT I3
4020 RETURN
4998 REM END SUBROUTINE 3'S
4999 REM ************************************************************************
5000 REM PRINT SUBROUTINE
5020 FOR I5=0 TO I0:ZS(I5)=NS(I5):NEXT
5040 IF I0=N THEN 5080
5050 FOR I5=I0+1 TO N
5060 REM:PRINT ZS(I5);
5070 NEXT
5080 REM:PRINT "I0=";I0
5090 GOSUB 6000
5990 RETURN
5998 REM END PRINT SUBROUTINE
5999 REM ************************************************************************
6000 REM SUBROUTINE TERM AND SYSTEM COMP
6010 TERM = 1:SUMWT = 0
6020 FOR I6= TO N
6030 SUMWT = SUMWT + ZS(I6)
6035 NEXT I6
6040 FOR 16= TO N
6050 TERM = TERM*ZS(I6)/SUMWT
6060 SUMWT = SUMWT -ZS(I6)
6070 NEXT I6
6080 PROB = PROB + TERM
6090 REM PRINT TERM, SUMWT, PROB
6990 RETURN
6998 REM END COMPUTE SUBROUTINE
6999 REM************************************************************************
```

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